60. **Proposed by Professor C. E. WHITE, Trafalgar, Indiana.**

Prove that every algebraic equation can be transformed into another equation of the same degree, but which wants its n^{th} term.

61. **Proposed by J. A. CALDERHEAD, A. B., Superintendent of Schools, Limaville, Ohio.**

Given $x^2+x\sqrt{xy}=10$, and $y^2+y\sqrt{xy}=20$ to find x and y by quadratics.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

40. **Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

The closed portion of the curve known as "The Cocked Hat," equation

$$x^4+x^2y^2+4ax^2y-2a^2x^2+3a^2y^2-4a^3y+a^4=0,$$

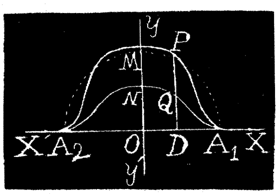
revolves around the axis of y. Find the *campanulate* volume generated. If the same portion of the curve revolve around the axis of x, find the *fusiform* volume generated. Also, determine the area of this closed portion of the curve.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Taxas; W. C. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts; and the PROPOSER.

Solving the equation for x^2 we get $x^2=\pm\frac{1}{2}y\sqrt{y^2+8ay}-\frac{1}{2}(y^2+4ay-2a^2)$.

$\therefore$ The campanulate volume generated by the area MPA_1QNM is

$$V=\frac{\pi}{2}\int_0^a(y\sqrt{y^2+8ay}-y^2-4ay+2a^2)dy+\frac{\pi}{2}\int_0^{\frac{1}{3}a}(y\sqrt{y^2+8ay}+y^2+4ay-2a^2)dy.$$



$$=\frac{\pi}{2}\Big[\tfrac{1}{3}(y^2+8ay)^{\frac{3}{2}}-2a(y-4a)\sqrt{y+8ay}+32a^3\log\Big\{y+4a+\sqrt{y^2+8ay}\Big\}-\tfrac{1}{3}y^3-2ay^2+2a^2y\Big]_0^a+\frac{\pi}{2}\Big[\tfrac{1}{3}y^3+2ay^2-2a^2y+\tfrac{1}{3}(y^2+8ay)^{\frac{3}{2}}-2a(y+4a)\sqrt{y^2+8ay}+32a^3\log\Big\{y+4a+\sqrt{y^2+8ay}\Big\}\Big]_0^{\frac{1}{3}a}$$

$$=\tfrac{4}{3}\pi a^3(12\log 3-13).$$

[ZERR, MATZ, and BLACK.]

From the equation we get $y=\dfrac{2a(a^2-x^2)\pm(a^2-x^2)\sqrt{a^2-x^2}}{x^2+3a^2}$.

$\therefore$ The fusiform volume is

$$V=8a\pi\int_0^a\frac{(a^2-x^2)^2\sqrt{a^2-x^2}}{(x^2+3a^2)^2}dx=8\pi a^3\int_0^{\frac{1}{2}\pi}\frac{\cos^6\theta d\theta}{(4-\cos^2\theta)^2},\text{ where } x=a\sin\theta,$$

$$=8\pi a^3\left[\frac{17\theta}{2}-\frac{44\sqrt{3}}{3\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\tan\theta\right)+\frac{\cos\theta\sin\theta}{2}+\frac{8\sin\theta\cos\theta}{9\cos^2\theta+12\sin^2\theta}\right]_0^{\frac{1}{2}\pi}$$

$$=4\pi^2a^3\left(\frac{17}{2}-\frac{44\sqrt{3}}{9}\right)=\frac{2}{9}\pi^2a^3(153-88\sqrt{3}).$$

Also area is $A=2\displaystyle\int_0^a\frac{(a^2-x^2)\sqrt{a^2-x^2}}{x^2+3a^2}dx=2a^2\int_0^{\frac{1}{2}\pi}\frac{\cos^4\theta d\theta}{4-\cos^2\theta}=2a^2\Bigg[\frac{8\sqrt{3}}{3}$

$\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\tan\theta\right)-\dfrac{9\theta}{2}-\dfrac{\sin\theta\cos\theta}{2}\Bigg]_0^{\frac{1}{2}\pi}=\dfrac{1}{6}\pi a^2(16\sqrt{3}-27).$

[ZERR, and MATZ.]

Or, fusiform volume$=2\pi\displaystyle\int_0^a(y_1^2-y_2^2)dx=16a\pi\int_0^a\frac{(a^2+x^2)^{\frac{5}{2}}dx}{(x^2+3a^2)^2}$

$$=16a^3\pi\int_0^{\frac{1}{2}\pi}\frac{\cos^6\theta d\theta}{(4-\cos^2\theta)^2}$$

$$=16a^3\pi\int_0^{\frac{1}{2}\pi}\left(\frac{\cos^6\theta}{4}+\frac{2\cos^8\theta}{4^3}+\frac{3\cos^{10}\theta}{4^4}+\frac{4\cos^{12}\theta}{4^5}+\ldots\ldots\right)d\theta$$

$$=8\pi^2a^3\left\{\frac{1.3.5}{2.4.6.4^2}+\frac{2.1.3.5.7}{2.4.6.8.4^3}+\frac{3.1.3.5.7.9}{2.4.6.8.10.4^4}+\ldots\ldots\right\},\text{ since}$$

$$\int_0^{\frac{1}{2}\pi}\cos^{4m}x dx=\frac{1.3.5\ldots\ldots(2m-1)}{2.4.6\ldots\ldots\ldots\ldots(2m)}\cdot\frac{\pi}{2}.$$

Area of closed portion$=2\displaystyle\int_0^a(y_1-y_2)dx+4\int_0^a\frac{(a^2-x^2)^{\frac{3}{2}}dx}{x^2+3a^2}$.

Let $x=a\sin\theta$, $A=4a^2\displaystyle\int_0^{\frac{1}{2}\pi}\frac{\cos^4\theta d\theta}{4-\cos^2\theta}=4a^2\int_0^{\frac{1}{2}\pi}\frac{\cos^4\theta}{4}+\frac{\cos^6\theta}{4^2}+\frac{\cos^8\theta}{4^3}$

$+\dfrac{\cos^{10}\theta}{4^4}+\ldots\ldots)d\theta=2a^2\pi\left\{\dfrac{1.3}{2.4.4}+\dfrac{1.3.5}{2.4.6.4^2}+\dfrac{1.3.5.7}{2.4.6.8.4^3}+\ldots\ldots\right\},$

which series is also convergent. [BLACK.]